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MASTER

AUTHOR(S): Dennis R. Peterson
Clarence M. Fowler

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LOS ALAMOS SCIENTIFIC LABORATORY

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PERTURBATION ANALYSIS OF RAIL GUNS POWERED BY EXPLOSIVE MAGNETIC FLUX COMPRESSION

by

D. R. Peterson
C. M. Fowler

ABSTRACT

Perturbation methods are used to predict the performance of rail guns powered by explosive magnetic flux compression, and the results are compared with experimental data. The problem of designing explosive magnetic flux compression generators for optimum rail gun performance is also discussed.

I. INTRODUCTION

Explosive magnetic flux compression generators ("explosive generators" for short) are pulsed electrical generators that are powered by chemical explosives. These devices have recently been used successfully to power rail guns.[1] Details of the explosive generators are given in a separate paper.[2] In the present paper, rail guns powered by explosive generators are analyzed using perturbation methods.[3]

II. THE GOVERNING EQUATIONS

Mathematically, an explosive generator may be represented in the rail gun circuit as a time-decreasing inductance $L_g(t)$ as shown in Fig. II.1.

P is a priming current source; S is a crowbar switch closed at time $t = 0$; I is current; R_A is the resistance of the rail gun armature; x is the projectile position; and L_g, R_y, L_c, R_c, L_r, and R_r are the inductance and resistance of the generator, the generator-to-rail gun coupling, and the rail gun.
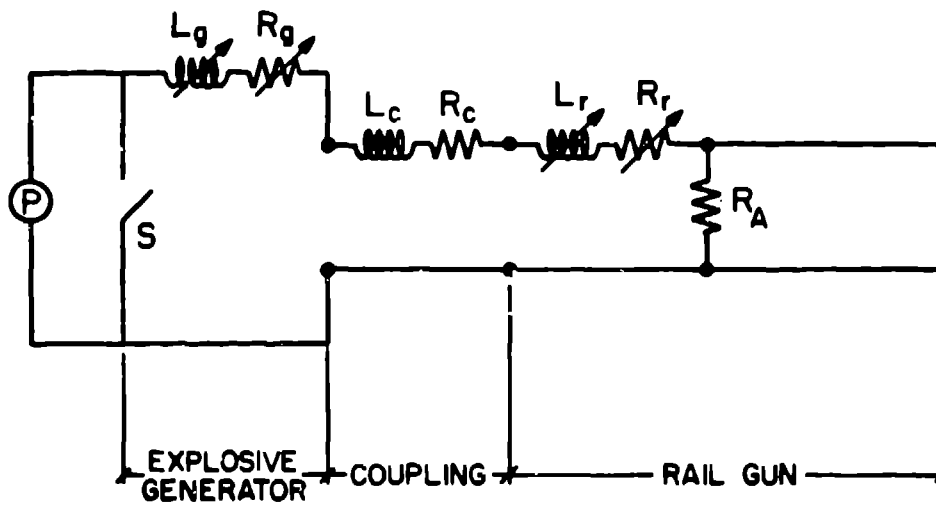
1

Fig. II.1. Schematic diagram of explosive generator
powered rail gun.

It is useful to write the time-decreasing inductance of the explosive generator as a quadratic in time.

$$L_g = L_0 - A_1 t - A_2 t^2, \quad 0 \le t \le t_B, \tag{II.1}$$

where L_0, A_1, and A_2 are constants, and t_B is the generator burn time.

The governing differential equations and initial conditions of the circuit and of the projectile motion are

$$\frac{d(LI)}{dt} + RI = 0, \tag{II.2}$$

$$\frac{d^2 x}{dt^2} = \frac{L'I^2}{2M} - \frac{f}{M}, \tag{II.3}$$

$$L\Big|_{t=0} = L_a, \tag{II.4}$$

$$I\Big|_{t=0} = I_a, \tag{II.5}$$

$$x\Big|_{t=0} = 0, \text{ and} \tag{II.6}$$

$$\frac{dx}{dt}\Big|_{t=0} = v_a, \tag{II.7}$$

where L is the total inductance and R is the total resistance of the rail gun circuit, L' is the inductance per unit length of the rail gun, M is the projectile mass, and f is the friction force opposing the projectile motion (taken as constant in the present analysis).

The current path through the explosive generator decreases linearly with time so that R_g may be written

$$R_g = R_g' (l_0 - ct), \tag{II.8}$$

3

where l_0 is the initial length of the generator, R_g' is the resistance per unit length of the generator, and c is the explosive detonation speed. The total circuit resistance R is given by

$$R = \Gamma_i - R_g'ct + R'x, \text{ and} \tag{II.9}$$

$$R_a = R_g'l_0 + R_c + R_A, \tag{II.10}$$

where R' is the resistance per unit length of the rail gun.

The instantaneous circuit inductance L is

$$L = L_a - A_1t - A_2t^2 + L'x, \text{ and} \tag{II.11}$$

$$L_a = L_0 + L_c. \tag{II.12}$$

With Eqs. (II.9) and (II.11), x may be eliminated from Eqs. (II.2) and (II.3):

$$\frac{d^2L}{dt^2} = \frac{L'^2I^2}{2M} - \frac{fL'}{M} - 2A_2, \tag{II.13}$$

$$\frac{d(LI)}{dt} + I[R_a - R_g'ct + \frac{R'}{L'}(L - L_a + A_1t + A_2t^2)] = 0, \tag{II.14}$$

$$L\Big|_{t=0} = L_a, \tag{II.15}$$

$$\frac{dL}{dt}\Big|_{t=0} = L'v_a - A_1, \text{ and} \tag{II.16}$$

$$I\Big|_{t=0} = I_a. \tag{II.17}$$

III. THE ALMOST-LINEAR GENERATOR

The case in which the quadratic term in Eq. (II.1) is small (for $0 \le t \le t_B$) is now considered.

4

It is convenient to define a dimensionless inductance p, a dimensionless current i, and a dimensionless time T:

$$p = L/L_b, \tag{III.1}$$

$$i = I/I_b, \tag{III.2}$$

$$T = t/k, \tag{III.3}$$

$$L_b = \frac{L_a}{1 + \frac{M}{L_a}\left(\frac{A_i - L'v_a}{I_a L'}\right)^2}, \tag{III.4}$$

$$I_b = L_a I_a / L_b, \text{ and} \tag{III.5}$$

$$k = (ML_b)^{1/2}/L'I_b. \tag{III.6}$$

In dimensionless form, Eqs. (II.13) to (II.17) are written

$$\frac{dq}{dT} = \frac{1}{2} i^2 - 2\delta_2 - \delta_3, \tag{III.7}$$

$$q = \frac{dp}{dT}, \tag{III.8}$$

$$\frac{d(pi)}{dT} + i[\delta_1 + \delta_0(p - p_a + P_4 T + \delta_2 T^2)] = 0, \tag{III.9}$$

$$p\Big|_{T=0} = p_a = (1 - q_a^2)^{-1}, \tag{III.10}$$

$$q\Big|_{T=0} = q_a, \tag{III.11}$$

$$i\Big|_{T=0} = 1/p_a, \tag{III.12}$$

5

$$\delta_0 = kR'/L', \tag{III.13}$$

$$\delta_1 = kR_{\bar{c}}/L_b, \tag{III.14}$$

$$\delta_2 = k^2 A_2/L_b, \tag{III.15}$$

$$\delta_3 = f/L'I_b^2, \tag{III.16}$$

$$B_4 = k(A_1 - R_g'L'c/R')/L_b, \tag{III.17}$$

$$p_a = L_a/L_b, \text{ and} \tag{III.18}$$

$$q_a = k(L'v_a - A_1)/L_b. \tag{III.19}$$

"uppose that p, q, and i (and their derivatives); T; and B_4 are of order unity; that $\delta_0 \ll 1$; and that δ_1, δ_2, and δ_3 are of order δ_0, and write for p, q, and i:

$$p = p_0 + \delta_0 p_1 + \delta_0^2 p_2 + \cdots , \tag{III.20}$$

$$q = q_0 + \delta_0 q_1 + \delta_0^2 q_2 + \cdots , \text{ and} \tag{III.21}$$

$$i = i_0 + \delta_0 i_1 + \delta_0^2 i_2 + \cdots , \tag{III.22}$$

where the p_i, q_i, and i_i are of order unity. Then,

$$pi = p_0 i_0 + \delta_0 (p_0 i_1 + p_1 i_0) + \cdots, \text{ and} \tag{III.23}$$

$$i^2 = i_0^2 + 2\delta_0 i_0 i_1 + \cdots . \tag{III.24}$$

In Eqs. (III.7) to (III.12), the coefficient of each power of δ_0 is independently zero. Retaining no term of order greater than δ_0,

6

$$\frac{dq_0}{dT} = \frac{1}{2} i_0^2, \tag{III.25}$$

$$\frac{dq_1}{dT} = i_0 i_1 - B_2, \tag{III.26}$$

$$\frac{d(p_0 i_0)}{dT} = 0, \tag{III.27}$$

$$\frac{d(p_0 i_1 + p_1 i_0)}{dT} + i_0(\delta_1/\delta_0 + p_0 - p_a + B_4 T) = 0, \tag{III.28}$$

$$p_0 \Big|_{T=0} = p_a, \tag{III.29}$$

$$p_1 \Big|_{T=0} = 0, \tag{III.30}$$

$$q_0 \Big|_{T=0} = q_a, \tag{III.31}$$

$$q_1 \Big|_{T=0} = 0, \tag{III.32}$$

$$i_0 \Big|_{T=0} = 1/p_a, \tag{III.33}$$

$$i_1 \Big|_{T=0} = 0, \text{ and} \tag{III.34}$$

$$B_2 = (2\delta_2 + \delta_3)/\delta_0. \tag{III.35}$$

The simultaneous differential equations may be solved in the order Eqs. (III.27), (III.25), (III.28), and (III.26):

$$i_0 = p_0^{-1}, \tag{III.36}$$

$$p_0 = f(q_0), \tag{III.37}$$

$$T = h(q_{\iota}) - h(q_a), \tag{III.38}$$

$$i_1 = -i_0^2 p_1 - i_0[h(q_0) + B_1 + B_3 g(q_0) + B_4 w(q_0)], \tag{III.39}$$

$$p_1 = q_0 X - u(q_0)Y, \tag{III.40}$$

$$q_1 = \frac{1}{2} i_0^2 \{X - [3h(q_0) + 2q_0 f^2(q_0)] Y\}, \tag{III.41}$$

$$X = X_0 + B_1 X_1 + B_2 X_2 + B_3 X_3 + B_4 X_4, \tag{III.42}$$

$$Y = Y_0 + B_1 Y_1 + B_2 Y_2 + B_3 Y_3 + B_4 Y_4, \tag{III.43}$$

$$X_n = x_n(q_0) - x_n(q_a), \quad n = 0, 1, 2, 3, 4, \tag{III.44}$$

$$Y_n = y_n(q_0) - y_n(q_a), \quad n = 0, 1, 2, 3, 4, \tag{III.45}$$

$$x_0(z) = 3v^2(z) + w(z), \tag{III.46}$$

$$x_1(z) = -g(z) + 3zv(z), \tag{III.47}$$

$$x_2(z) = \frac{1}{2} z f^2(z) - \frac{15}{4} h(z) + 3f(z)g(z), \tag{III.48}$$

$$x_3(z) = g^2(z) - 3g^2(z)/f(z), \tag{III.49}$$

$$x_4(z) = h(z) + 3g(z) [v(z)-1] + g^3(z) [\frac{2}{3} - 3/f(z)], \tag{III.50}$$

$$y_0(z) = g(z) + zv(z), \tag{III.51}$$

$$y_1(z) = z^2, \tag{III.52}$$

$$y_2(z) = f(z), \tag{III.53}$$

$$y_3(z) = z - g(z)/f(z), \tag{III.54}$$

$$y_4(z) = 2v(z) - g^2(z)/f(z), \tag{III.55}$$

$$f(z) = (1-z^2)^{-1}, \tag{III.56}$$

$$g(z) = \frac{1}{2} \log \left(\frac{1+z}{1-z}\right), \tag{III.57}$$

$$h(z) = g(z) + zf(z), \tag{III.58}$$

$$u(z) = 3v(z) + f(z), \tag{III.59}$$

$$v(z) = zg(z) - 1, \tag{III.60}$$

$$w(z) = g^2(z) + f(z), \tag{III.61}$$

$$B_1 = -h(q_a) - B_3 g(q_a) - B_4 w(q_a), \text{ and} \tag{III.62}$$

$$B_3 = 2[\delta_1/\delta_0 - p_a - B_1 h(q_a)]. \tag{III.63}$$

To summarize, a solution

$$p = p_0 + \delta_0 p_1, \tag{III.64}$$

$$q = q_0 + \delta_0 q_1, \text{ and} \tag{III.65}$$

$$i = i_0 + \delta_0 i_1, \tag{III.66}$$

to order δ_0 has been obtained to the dimensionless Eqs. (III.7) to (III.12), using perturbation methods.

Dimensional quantities, such as the rail gun current and the projectile position and velocity, may be calculated from the dimensionless variables p, q, and i:

$$I = I_b i, \tag{III.67}$$

9

$$x = [L_b(p-p_a) + A_1 t + A_2 t^2] /L', \text{ and} \tag{III.68}$$

$$\frac{dx}{dt} = \left(\frac{L_b}{k}q + A_1 + 2A_2 t\right)/L'. \tag{III.69}$$

Equations (III.36) to (III.41) are expressed in terms of the variable q_0. Newton's method may be used to calculate by iteration q_0 at any time t,

$$Q_{n+1} = Q_n + \frac{kt + h(q_a) - h(Q_n)}{2f^2(Q_n)} , \tag{III.70}$$

where Q_n is the nth iteration of q_0.

Fig. III.1 compares an experimentally measured rail gun current with a numerically calculated current and with a current calculated with Eq. (III.67). Fig. III.2 compares numerically calculated rail gun projectile position and velocity with position and velocity calculated with Eqs. (III.68) and (III.69). The parameters for the numerical and perturbation solution are listed in Table III.1. The perturbation solution tracks the numerical solution for about 200 µs despite the fact that the parameters δ_0 and δ_1 are of order unity.

IV. THE NEAR-OPTIMUM QUADRATIC GENERATOR

The case of near-constant current during the generator burn will now be considered. Define a dimensionless inductance F, a dimensionless current H, and dimensionless time S,

$$F = L/L_a, \tag{IV.1}$$

$$G = \frac{dF}{dS} , \tag{IV.2}$$

$$H = I/I_a, \tag{IV.3}$$

$$S = \omega t, \text{ and} \tag{IV.4}$$

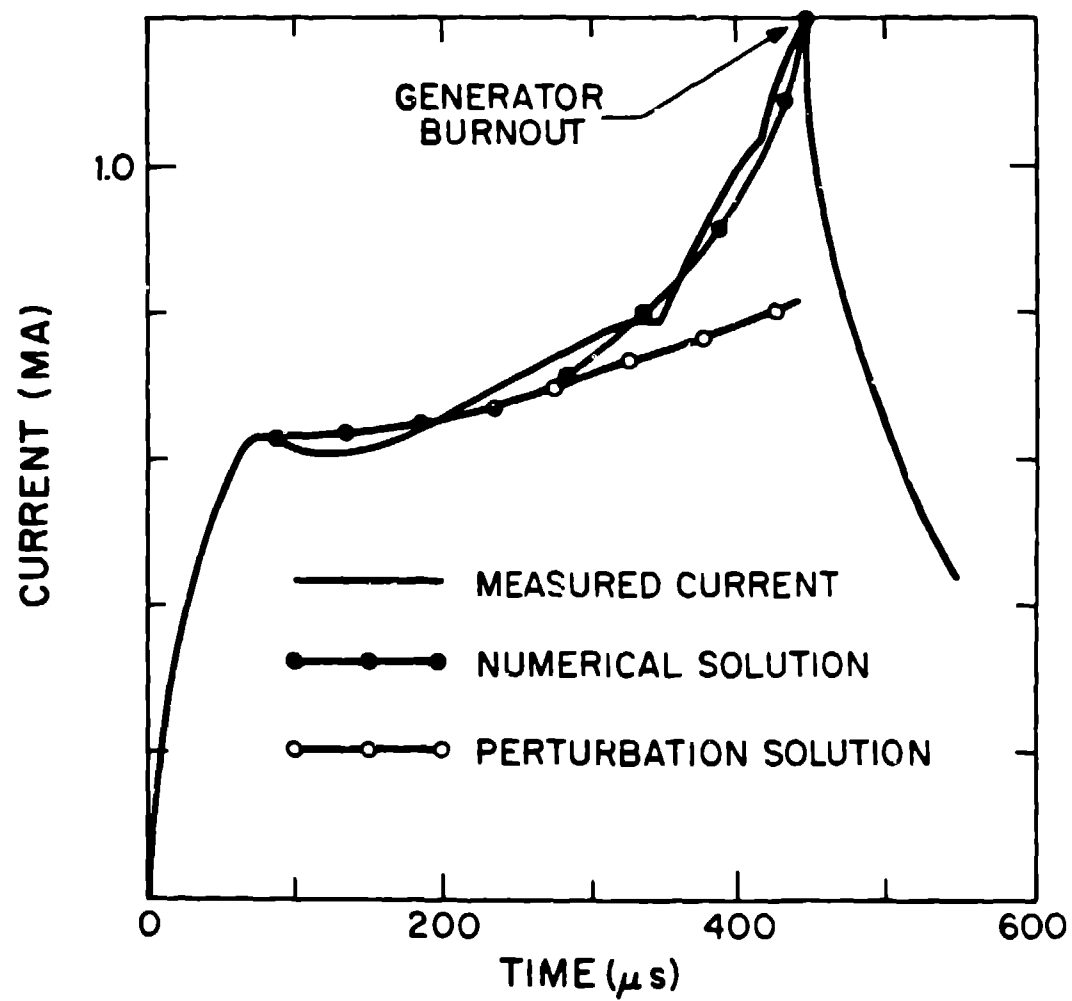$$\omega = L'I_a(L_a M)^{-1/2}. \tag{IV.5}$$

Fig. III.1. Comparison of experimentally measured rail gun current with numerically calculated current and current calculated with Eq. (III.67).
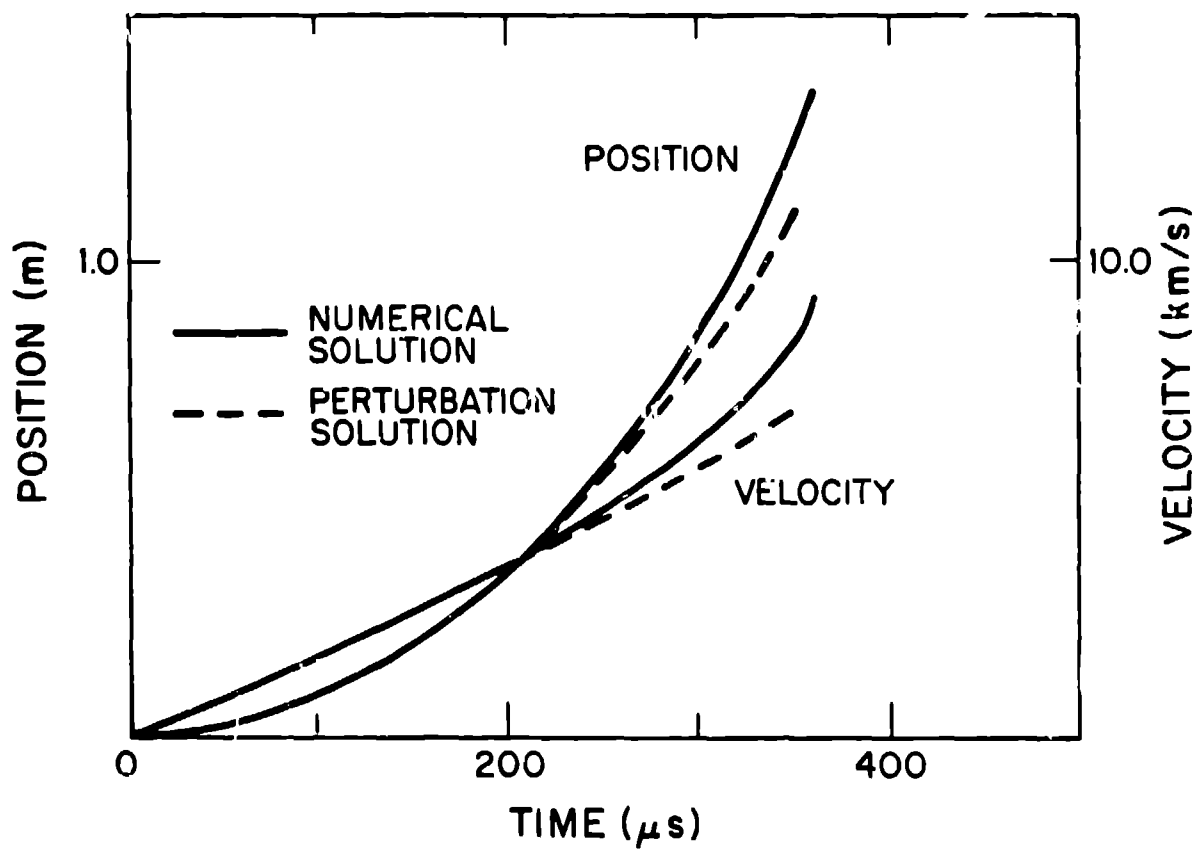
Fig. III.2. Comparison of rail gun projectile position and velocity calculated numerically and with Eqs. (III.68) and (III.69).

12

TABLE III.1

PARAMETERS USED FOR NUMERICAL AND PERTURBATION SOLUTIONS
SHOWN IN FIGS. III.1 AND III.2

L_0	1.4×10^{-6} H
L_c	2.4×10^{-7} H
L'	0.25×10^{-6} H/m
R_c	0 ohms
R_A	0.001 ohms
R'	9.7×10^{-4} ohms/m
R_g'	3.5×10^{-4} ohms/m
l_0	2.44 m
I_a	6.4×10^5 A
A_1	1.74×10^{-3} H/s
A_2	5.52 H/s^2
V_a	65 m/s
M	3.1 g
f	0
c	6600 m/s
q_a	-0.3989
δ_0	1.4351
δ_1	0.4291
B_2	0.6590
B_4	0.2649

In dimensionless form, the differential equations and initial conditions Eqs. (II.13) to (II.17) are written

$$\frac{d^2F}{dS^2} = \frac{1}{2}(H^2-1) - \varepsilon_2, \tag{IV.6}$$

$$\frac{d(FH)}{dS} + \varepsilon_0 H (F-1+D_3+D_4S+D_5S^2) = 0, \tag{IV.7}$$

$$F(0) = 1, \tag{IV.8}$$

$$G(0) = -\varepsilon_1, \tag{IV.9}$$

$$\frac{d^2F_0}{dS^2} = \frac{1}{2} (H_0{}^2 - 1), \qquad\qquad\qquad\qquad (IV.20)$$

$$\frac{d^2F_1}{dS^2} = H_0 H_1 - D_2, \qquad\qquad\qquad\qquad (IV.21)$$

$$\frac{d(F_0 H_0)}{dS} = 0, \qquad\qquad\qquad\qquad (IV.22)$$

$$\frac{d(F_0 H_1 + F_1 H_0)}{dS} + H_0 (F_0 - 1 + D_3 + D_4 S + D_5 S^2) = 0, \qquad\qquad (IV.23)$$

$$F_0(0) = 1, \qquad\qquad\qquad\qquad (IV.24)$$

$$F_1(0) = 0, \qquad\qquad\qquad\qquad (IV.25)$$

$$G_0(0) = 0, \qquad\qquad\qquad\qquad (IV.26)$$

$$G_1(0) = -D_1, \qquad\qquad\qquad\qquad (IV.27)$$

$$H_0(0) = 1, \qquad\qquad\qquad\qquad (IV.28)$$

$$H_1(0) = 0, \qquad\qquad\qquad\qquad (IV.29)$$

$$D_1 = \epsilon_1/\epsilon_0, \text{ and} \qquad\qquad\qquad\qquad (IV.30)$$

$$D_2 = \epsilon_2/\epsilon_0. \qquad\qquad\qquad\qquad (IV.31)$$

The simultaneous differential equations may be solved in the order (IV.22), (IV.20), (IV.23), and (IV.21).

$$F_0 = 1, \qquad\qquad\qquad\qquad (IV.32)$$

$$G_0 = 0, \qquad\qquad\qquad\qquad (IV.33)$$

$$H(0) = 1, \tag{IV.10}$$

$$\epsilon_0 = \frac{R'}{\omega L'} , \tag{IV.11}$$

$$\epsilon_1 = \frac{L'}{\omega L_a^2} (A_1/L'-v_a), \tag{IV.12}$$

$$\epsilon_2 = \frac{M}{L'I_a^2} (2A_2/L'-L'I_a^2/2M+f/M), \tag{IV.13}$$

$$D_3 = L'R_a/L_aR', \tag{IV.14}$$

$$D_4 = A_1/L_a\omega - L'R_g'c/\omega L_aR', \text{ and} \tag{IV.15}$$

$$D_5 = A_2/L_a\omega^2. \tag{IV.16}$$

In the absence of flux losses, setting the parameters ϵ_1 and ϵ_2 to zero would result in a constant inductance, hence, constant current, rail gun circuit; the increasing inductance of the rail gun is exactly offset by the declining inductance of the explosive generator.

Suppose that F, G, and H (and their derivatives), S, D_3, D_4, and D_5 are of order unity; $\epsilon_0 \ll 1$; and ϵ_1 and ϵ_2 are of order ϵ_0. Write for F, G, and H,

$$F = F_0 + \epsilon_0 F_1 + \epsilon_0^2 F_2 + \dots , \tag{IV.17}$$

$$G = G_0 + \epsilon_0 G_1 + \epsilon_0^2 G_2 + \dots , \text{ and} \tag{IV.18}$$

$$H = H_0 + \epsilon_0 H_1 + \epsilon_0^2 H_2 + \dots . \tag{IV.19}$$

In Eqs. (IV.6) to (IV.10), the coefficient of each power of ϵ_0 is independently zero. Retaining only terms of order ϵ_0,

$$H_0 = 1,$$
(IV.34)

$$F_1 = -2D_5(\sin S - S) - (D_4 - D_2)(\cos S - 1) - (D_1 - D_3)\sin S$$
$$- D_3 S - \frac{1}{2}D_4 S^2 - \frac{1}{3}D_5 S^3,$$
(IV.35)

$$G_1 = -2D_5(\cos S - 1) + (D_4 - D_2)\sin S - (D_1 - D_3)\cos S$$
$$- D_3 - D_4 S - D_5 S^2, \text{ and}$$
(IV.36)

$$H_1 = 2D_5(\sin S - S) + (D_4 - D_2)(\cos S - 1) + (D_1 - D_3)\sin S.$$
(IV.37)

To summarize, a solution

$$F = F_0 + \epsilon_0 F_1,$$
(IV.38)

$$G = G_0 + \epsilon_0 G_1, \text{ and}$$
(IV.39)

$$H = H_0 + \epsilon_0 H_1,$$
(IV.40)

to order ϵ_0 has been obtained to the dimensionless Eqs. (IV.20) to (IV.23).

Dimensional quantities, such as the rail gun current and the projectile position and velocity, may be calculated from the dimensionless variables F, G, and H,

$$I = I_a H,$$
(IV.41)

$$x = [L_a(F-1) + A_1 t + A_2 t^2]/L', \text{ and}$$
(IV.42)

$$\frac{dx}{dt} = (\omega L_a G + A_1 + 2A_2 t)/L'.$$
(IV.43)

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